N13/5/MATHL/HP3/ENG/TZ0/SP

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MATHEMATICS
HIGHER LEVEL
PAPER 3 – STATISTICS AND PROBABILITY

Tuesday 19 November 2013 (afternoon)

1 hour

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the ***Mathematics HL and Further Mathematics SL*** *information booklet* is required for this paper.
- The maximum mark for this examination paper is *[60 marks]*.

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4 pages

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. *[Maximum mark: 8]*

A traffic radar records the speed, v kilometres per hour (km h^{-1}), of cars on a section of a road. The following table shows a summary of the results for a random sample of 1000 cars whose speeds were recorded on a given day.

Speed	**Number of cars**
$50 \le v < 60$	5
$60 \le v < 70$	13
$70 \le v < 80$	52
$80 \le v < 90$	68
$90 \le v < 100$	98
$100 \le v < 110$	105
$110 \le v < 120$	289
$120 \le v < 130$	142
$130 \le v < 140$	197
$140 \le v < 150$	31

(a) Using the data in the table,

(i) show that an estimate of the mean speed of the sample is $113.21\ \text{km h}^{-1}$;

(ii) find an estimate of the variance of the speed of the cars on this section of the road. *[4]*

(b) Find the 95 % confidence interval, I, for the mean speed. *[2]*

(c) Let J be the 90 % confidence interval for the mean speed.

Without calculating J, explain why $J \subset I$. *[2]*

2. *[Maximum mark: 14]*

A farmer is selling apples and oranges. The weights X and Y, in grams, of the apples and oranges respectively are normally distributed with $X \sim N(180, 14^2)$ and $Y \sim N(150, 12^2)$.

(a) Find the probability that the weight of a randomly chosen apple is more than 1.5 times the weight of a randomly chosen orange. *[7]*

(b) Katharina buys 4 apples and 6 oranges. Find the probability that the total weight is greater than 1.5 kilograms. *[7]*

3. *[Maximum mark: 14]*

Jenny tosses seven coins simultaneously and counts the number of tails obtained. She repeats the experiment 750 times. The following frequency table shows her results.

Number of tails	**Frequency**
0	6
1	19
2	141
3	218
4	203
5	117
6	38
7	8

(a) It is claimed that all of these seven coins are fair and it is decided to test this claim using a suitable χ^2 test.

(i) State the null and alternative hypotheses.

(ii) State a decision rule at the 5% level of significance.

(iii) Find the value of the test statistic.

(iv) Write down your conclusion. *[10]*

(b) Explain what can be done with this data to decrease the probability of making a type I error. *[2]*

(c) (i) State the meaning of a type II error.

(ii) Write down how to proceed if it is required to decrease the probability of making both a type I and type II error. *[2]*

4. *[Maximum mark: 10]*

Francisco and his friends want to test whether performance in running 400 metres improves if they follow a particular training schedule. The competitors are tested before and after the training schedule.

The times taken to run 400 metres, in seconds, before and after training are shown in the following table.

Competitor	**A**	**B**	**C**	**D**	**E**
Time before training	75	74	60	69	69
Time after training	73	69	55	72	65

Apply an appropriate test at the 1 % significance level to decide whether the training schedule improves competitors' times, stating clearly the null and alternative hypotheses. (It may be assumed that the distributions of the times before and after training are normal.)

5. *[Maximum mark: 14]*

Let X and Y be independent random variables with $X \sim P_o(3)$ and $Y \sim P_o(2)$.

Let $S = 2X + 3Y$.

(a) Find the mean and variance of S. *[2]*

(b) Hence state with a reason whether or not S follows a Poisson distribution. *[2]*

Let $T = X + Y$.

(c) Find $\mathrm{P}(T = 3)$. *[4]*

(d) Show that $\mathrm{P}(T = t) = \sum_{r=0}^{t} \mathrm{P}(X = r)\mathrm{P}(Y = t - r)$. *[2]*

(e) Hence show that T follows a Poisson distribution with mean 5. *[4]*